PRACTICAL CONSIDERATIONS FOR DETERMINING POLARIZATION PROPERTIES FROM MEASURED LINEAR COMPONENTS

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ABSTRACT

Polarization properties (e.g. axial ratio, sense, and tilt) of an antenna under test (AUT) are often calculated from measurements with a linear (or dual-linear) polarized range antenna. At first, these calculations appear to be simple and straightforward. However, there are several different conventions used in the literature and some important practical aspects of the measurements are often omitted. Neglect of these small details can easily lead to incorrect results, with the most common error being the reversal of the right-hand-circular and left-hand-circular polarization components. We note the differences in the published polarization conventions and provide practical tips for good polarization measurement practices. We also describe step-by-step procedures for determining AUT polarization properties from two styles of polarization measurements using a linear (or dual-linear) polarized range antenna.

Keywords: Polarization Components, Polarization Measurement, Calibration

1. Introduction

The motivation for this paper stemmed from the observation that there is often confusion among antenna engineers about how to adapt published polarization equations into practical techniques that can be used to reliably determine polarization properties of an antenna under test (AUT). Reliability in the determination of sense is the primary concern here, as mistakes in determining the dominant circularly polarized (CP) component of an AUT are surprisingly common in the industry. In some cases these mistakes are the result of misinterpreting published polarization equations due to time convention or coordinate system differences. We have found that it's more common for the mistakes to occur due to the subtle details surrounding the actual measurement and calibration process which are rarely discussed in publications. This paper attempts to fill that gap with practical details about two common polarimetric measurement techniques.

2. Conventions

We call the non-AUT the "range antenna" in this paper. This term is meant to represent any nominally linearly polarized antenna in any kind of range in either a transmitting or receiving configuration.

The IEEE convention of right-hand (RH) and left-hand (LH) polarization is used here, as is the case in all antenna polarization publications known to us. This convention dictates that the thumb is pointed in the direction of propagating energy (i.e. toward the destination), and the fingers are curled to match the rotation of the electric field vector, i.e. from the phase-leading component toward the phase-lagging component along the shortest angular path between them [1, 2].

The time convention of $e^{+j\omega t}$ is also used here, corresponding to the convention employed by the majority of modern commercial antenna measurement systems and VNAs. Although this is the most prevalent time convention, it is often not the convention assumed in published literature and is therefore a common cause of confusion. In the chosen $e^{+j\omega t}$ convention, +j indicates a phase-leading component, –j indicates a phase-lagging component, and e^{-jkr} represents phase change vs. radial propagation distance [2]. More discussion of time conventions is contained in Section 7.

Our coordinate system always has +z pointing out of the AUT and refers to a pair of orthogonal linear measurements as Horizontal "H-pol" and Vertical "V-pol" since that is the most common dual-pol range configuration. The reader should note that these nomenclatures are arbitrary and all H & V subscripted variables in this paper can be substituted with any two orthogonal linear polarizations as long as the technique for determining the total field is preserved.

An AUT roll-over-azimuth positioner with the roll axis coincident with the AUT z-axis is assumed. This common positioner configuration makes positioner roll equivalent to the ϕ spherical coordinate system position so the axis labels "Roll" and "ϕ" are used interchangeably.

3. Circularly Polarized Unit Vectors and Components

The well-known formulas for AUT polarization properties are

$$\text{Axial ratio} = 20\log\left|\frac{1+|\rho_c|}{1-|\rho_c|}\right| \text{dB} \quad (1)$$

$$\text{Tilt angle} = 0.5\arg(\rho_c) \quad (2)$$

$$\text{Sense: } \left.\begin{array}{l} |\rho_c| > 1 \text{ implies right-hand polarization} \\ |\rho_c| = 1 \text{ implies linear polarization} \\ |\rho_c| < 1 \text{ implies left-hand polarization} \end{array}\right\} \quad (3)$$

$$\rho_c = E_R / E_L \,, \qquad (4)$$

where E_R and E_L are the RH and LH circular polarization components, respectively [3]. It should be noted that some references use the inverse definitions of the complex circular polarization ratio ρ_c (i.e. with the LH component in the numerator) [4, 5]. This is a minor inconsistency that is straightforward to deal with. The confusion most often arises in determining the proper values for E_R and E_L in (4). More specifically, E_R and E_L are often swapped inadvertently, causing a predominantly RH antenna to appear LH and vice versa.
One reason for this common error relates to the subtle difference between the RH/LH unit vectors that create the CP polarization basis and the RH/LH components that are needed for (4). These quantities are related by conjugation, which is a mathematical necessity with a physical reason: to determine the RH/LH response (i.e. component) of an AUT, the incident field must be correspondingly RH/LH, respectively. Because the incident field +z direction is opposite that of the AUT, conjugation is necessary. Understanding how to properly apply this conjugation is the first step in proper determination of the AUT's polarization properties. For a practical example of why conjugation is necessary (and why a RH antenna both transmits and receives RH), consider the simple microstrip patch antenna depicted in Figure 1.

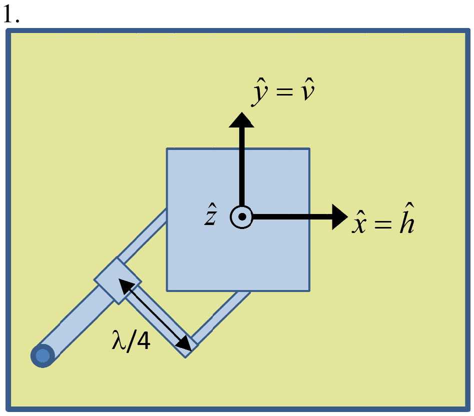

Figure 1 – Microstrip Patch Antenna

In this example, the visible feed network clearly indicates that the antenna is intended to be RH. Considered in transmitting mode, energy applied to the feed port will lead in the $\hat{x}$ polarization and lag in the $\hat{y}$ polarization, creating predominantly RH radiation. This concept leads us to construct the ideal RH unit vector for this case, recalling that –j indicates lagging phase:

$$\hat{u}_R = \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \qquad (5) \quad [3].$$

Now we measure this transmitting antenna in an antenna range with a linear (or dual-linear) receiving range antenna with pre-established $\hat{h}$ (horizontal) and $\hat{v}$ (vertical) reference directions as shown in Figure 1. Since the range reference unit vectors $\hat{h}$ and $\hat{v}$ have been arbitrarily chosen to correspond with the AUT's $\hat{x}$ and $\hat{y}$ unit vectors, respectively, the vertical measured[1] component E_V will lag the horizontal measured component E_H when the AUT is in the orientation shown in Figure 1. To properly determine the RH component E_R of the radiation produced by the AUT it is necessary to sum these two orthogonal polarization measurements in a way that counteracts the ideal RH phase relationship. Conjugation accomplishes that task. We first establish the total field

$$E_T = E_H \hat{h} + E_V \hat{v} = E_H \hat{x} + E_V \hat{y} \qquad (6)$$

and then find the RH component with a dot product of the complex vectors, which mathematically requires a conjugation of the unit vector [3]:

$$E_R = E_T \bullet \hat{u}_R^* \qquad (7)$$

$$E_R = \frac{E_H + jE_V}{\sqrt{2}} \qquad (8)$$

It can be seen from (8) that the RH component is determined by placing a phase *lead* on the v-/y-pol component, effectively counteracting the phase lag imparted by the AUT feed network. This is analogous to what occurs in the same RH AUT when in receiving mode. Consider the reciprocal measurement case where the antenna in Figure 1 is a receiving AUT. In this case, the antenna is illuminated by two orthogonal linear polarizations with a common phase reference. The feed network will impart a phase lag on the v-/y-pol energy compared with the h-/x-pol energy. To determine the proper RH component of the AUT, it is therefore necessary to advance the phase of the v-/y-pol energy, effectively creating a RH incident field and yielding the same result obtained in (8) for the transmitting example. For completeness, the LH component in both cases is

[1] E_V and E_H represent the complex linear voltage measurements reported by the antenna measurement system. These values are directly proportional to electric fields and in a system that has been calibrated to measure absolute gain they represent gain voltages.

$$E_L = \frac{E_H - jE_V}{\sqrt{2}}, \qquad (9)$$

which, as expected, is simply the conjugate equation (but not the conjugate value) of the RH component.

4. Establishing Range Reference Polarization Vectors

The discussion in the previous section contains similar information to what is already in the literature [3, 6]. However, there is little discussion in the literature regarding the antenna range reference polarization vectors $\hat{h}$ and $\hat{v}$. We have found that misunderstandings about how to properly establish these vectors are another common cause of incorrectly swapping the RH and LH components. In general, these vectors must be established through a calibration procedure. In some ranges this type of calibration procedure is performed daily, in others it is performed quite infrequently. A standard gain horn (SGH) with ideal[2] linear polarization in place of the AUT will be assumed as the calibration reference antenna throughout this discussion, although these principles apply to any linear calibration standard. A typical calibration procedure would consist of measuring a SGH at two orthogonal roll positions in the test zone. Data collected during this measurement are then used to calibrate the measurement system. Four important requirements are often placed on the calibration process:

1. The calibration data are collected in a representative chamber state (i.e. the SGH is measured where the AUT will be located, in the same chamber configuration in which the AUT will be measured, and preferably close in time to when the AUT will be measured)
2. The two polarization reference vectors must be orthogonal in space (i.e. 90° of rotation separates them)
3. The two polarization reference vectors must be established in the same plane (i.e. the same physical distance from the range antenna)
4. The RF subsystem connected to the range antenna is not modified between calibration and test

Adhering to these requirements provides a quality set of calibration data that is used to equalize the amplitude and phase for two orthogonal polarizations, including an absolute gain reference when gain substitution (also called "gain comparison") is employed [1, 7]. Equalizing the amplitude between the orthogonal polarizations insures $\hat{h}$ and $\hat{v}$ are indeed unit vectors of equal magnitude. Equalizing the phase between the orthogonal measurements insures that $\hat{h}$ and $\hat{v}$ share a common phase reference.

The last remaining calibration detail is subtle but critical for accurately differentiating between the LH and RH components in (4). Since the calibration process equalizes the phase between the two orthogonal polarizations, *it is effectively setting the direction of the polarization reference vectors* $\hat{h}$ *and* $\hat{v}$. Carefully tracking the direction of these vectors is the key to properly distinguishing between the RH and LH components. Referring back to the example of section 3, it is easy to see how a reversal of one of the reference vectors will impart a sign change in (6) that will propagate into (8) & (9), effectively swapping the RH and LH components. Our experience is that the dominant cause of improper AUT sense determinations is caused by a general lack of understanding of this concept: *The orientation of the SGH during calibration sets the direction of the range polarization reference vectors*. Therefore it is critical that the SGH orientations used during calibration are standardized and documented for each antenna range, and preferably across organizations. Measured data transferred between ranges must be accompanied by polarization reference vector information to avoid mistakes in interpreting AUT sense.

The four possible two-position calibration combinations are shown in Figure 2 with the assumption that positive roll (ϕ) motion rotates the SGH clockwise when viewed from the range antenna.

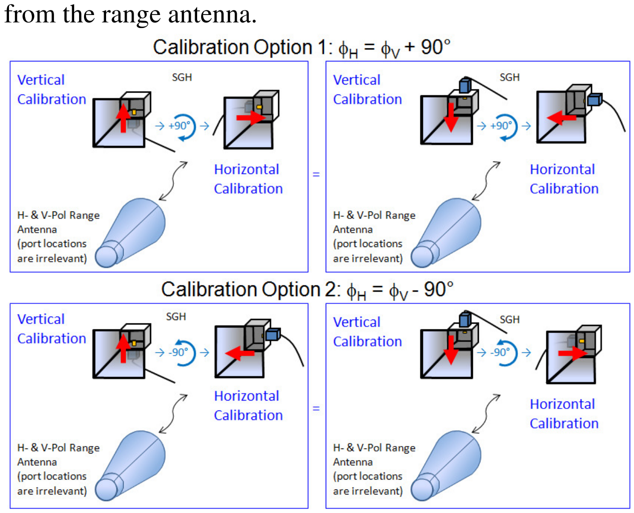

Figure 2 – Calibration Combinations

The red arrows in Figure 2 illustrate the reference vector directions established by each combination. Based on the arbitrary nature of drawing the polarization reference vector (i.e. it's equivalent to draw the vector pointing either out of the SGH port or into it as long as the chosen

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[2] The term "ideal" indicates infinite axial ratio with a known tilt angle that can be oriented perfectly during calibration

convention is maintained between the two SGH roll positions) and the arbitrary nature of choosing "lead" or "lag", the four combinations shown in Figure 2 yield two possibilities, each with two variants, for calculating RH and LH components as follows:

Calibration Option 1: $\phi_H = \phi_V + 90°$

$$E_R = \frac{E_H + jE_V}{\sqrt{2}};\ E_L = \frac{E_H - jE_V}{\sqrt{2}} \qquad (10;\ 11)$$

Or

$$E_R = \frac{E_V - jE_H}{\sqrt{2}};\ E_L = \frac{E_V + jE_H}{\sqrt{2}} \qquad (12;\ 13)$$

Calibration Option 2: $\phi_H = \phi_V - 90°$

$$E_R = \frac{E_V + jE_H}{\sqrt{2}};\ E_L = \frac{E_V - jE_H}{\sqrt{2}} \qquad (14;\ 15)$$

Or

$$E_R = \frac{E_H - jE_V}{\sqrt{2}};\ E_L = \frac{E_H + jE_V}{\sqrt{2}} \qquad (16;\ 17)$$

These equations have been verified to be accurate given the conventions and assumptions described above. It is strongly recommended that each of these conventions and assumptions be validated on a case-by-case basis prior to using these equations as-is, for any single difference will swap the equations for E_R and E_L. The complete procedure for determining the RH and LH components can be summarized as follows:

1. Verify measurement system time convention.
2. Assign a coordinate system with +z pointing out of the AUT as shown in Figure 1 and construct the RH AUT unit vector as shown in (5) by pointing your thumb in the +z direction, recalling that +j is leading phase and –j is lagging phase for an $e^{+j\omega t}$ convention. The LH unit vector is the conjugate of the RH unit vector.
3. Construct the total field vector in the range as shown in (6) with consideration for the SGH calibration positions as shown in Figure 2 to relate the range $\hat{h}$ and $\hat{v}$ unit vectors established during calibration to the chosen AUT $\hat{x}$ and $\hat{y}$ unit vectors from step 2.
4. Perform a dot product between the two vectors, remembering to conjugate the unit vector as shown in (7).

As demonstrated in section 3, all the equations and methods herein are valid whether the AUT is transmitting or receiving if the AUT is a reciprocal antenna. We find that it is more intuitive to always establish the RH and LH unit vectors by considering the AUT as transmitting. Although this technique is derived at AUT boresight with Cartesian coordinates, the method is applicable anywhere on the AUT pattern and in any coordinate system, e.g. ($\hat{r}$, $\hat{\theta}$, $\hat{\phi}$) spherical coordinates.

5. AUT Measurement with Linear Polarized Range Antenna and Linear Polarized SGH

The general "Method A" phase-amplitude antenna measurements performed with a linearly polarized (LP) range antenna calibrated with a LP SGH as described in [7] will be subdivided here to describe two variants denoted "Method A_1" and "Method A_2". This division is made to address the specifics of measurements made by a linear range antenna from those made by a dual-linear range antenna.

Method A_1: Single-Linear Range Antenna, Linear SGH

This section applies to the commonly-used measurement technique of using a single-linear antenna as the range antenna which is rotated 90° to obtain two orthogonal linear polarizations. This Method A_1 measurement system is distinct from Method A_2 in that there is no combination from separate RF paths to obtain CP components, only combinations of complex voltage measured through the same path. Although in theory only a single calibration measurement should be sufficient for Method A_1 measurements, two calibration measurements are generally performed due to differences that can arise from a number of error sources such as room reflections, range antenna misalignment, and rotary joint/flexing cable errors. As described previously, the polarization vectors are set by the orientations of the SGH during the calibration process. Just as with the calibration antenna, it is important to maintain consistency in the rotation of the range antenna positions between calibration and test to avoid reversing the polarization reference vectors.

A special case of a Method A_1 measurement involves using a linear polarization reference antenna (often a SGH) to measure the polarization properties of the range illumination in the quiet zone. In this case, the range antenna (really the illumination wavefront in the test zone that comprises the range antenna radiation properties, any compact range reflectors, stray energy, leakage, etc.) becomes the AUT. For a dual-linear range feed, the ports are evaluated separately and therefore each of the two measurements falls into this Method A_1 category.

Method A_2: Dual-Linear Range Antenna, Linear SGH

This section discusses the common practice of making two orthogonal linear measurements with a dual-linear polarized range antenna. The key differentiator of Method A_2 measurements, compared to Method A_1

measurements, is that the Method A_2 requires the complex combination of measurements from two separate RF paths, with the selection typically made by a high-speed polarization switch to allow for near-simultaneous measurement of both paths. Because it is uncommon for the two paths to be perfectly amplitude and phase matched a calibration is typically necessary for Method A_2 measurements to be meaningful.

We have noticed that several misunderstandings about the direction of the polarization reference vectors in Method A_2 measurements are prevalent in industry. Contrary to popular belief, it is not important to pay attention to the direction of the ports on a dual-pol range antenna, the orientation of the calibration antenna port with respect to those ports, or to how many compact range reflectors are in the propagation path. As described previously, the key is the relationship between the two calibration positions. This claim was confirmed by applying corrections from each of the 4 calibration combinations shown in Figure 2 to a Method A_2 measurement of a known RH AUT with Axial Ratio of approximately 7.8 dB. The results of applying (10) – (13) to each combination are shown in Table 1.

Table 1 – CP Component Magnitudes for Each Calibration Combination of a RH AUT

	Cal. Option 1: $\phi_H = \phi_V + 90°$		Cal. Option 2: $\phi_H = \phi_V - 90°$	
	$\phi_H = +90°$, $\phi_V = 0°$	$\phi_H = -90°$, $\phi_V = 180°$	$\phi_H = +90°$, $\phi_V = 180°$	$\phi_H = -90°$, $\phi_V = 0°$
$20 \cdot LOG_{10}(\lvert(E_H+jE_V)/\sqrt{2}\rvert)$	5.73	5.78	-1.76	-1.76
$20 \cdot LOG_{10}(\lvert(E_H-jE_V)/\sqrt{2}\rvert)$	-1.84	-1.65	5.80	5.78
$20 \cdot LOG_{10}(\lvert(E_V+jE_H)/\sqrt{2}\rvert)$	-1.84	-1.65	5.80	5.78
$20 \cdot LOG_{10}(\lvert(E_V-jE_H)/\sqrt{2}\rvert)$	5.73	5.78	-1.76	-1.76

The results in Table 1 demonstrate how (10) – (13) apply to Calibration Option 1 of Figure 2 and (14) – (17) apply to Calibration Option 2 of Figure 2.

6. Verification Method

Following the procedure in section 4 carefully is expected to remove all ambiguity about which circular component is RH and which is LH. To further boost confidence in the results, there is a simple verification that can be performed in the direction of the AUT pattern that coincides with a rotational axis of the AUT positioner, which often represents a boresight roll. Acknowledging that the source antenna will not be perfectly linear in practice, the sense of the polarization product between the source and AUT can be determined by observing the phase slope of a collection with sufficient angular resolution and span. Assuming an $e^{+j\omega t}$ convention, a negative phase slope vs. positive[3] roll (ϕ) indicates RH polarization, with a slope of exactly -1 phase degree/rotation degree corresponding to a perfect RH circular polarization product. This behavior is evident in both low AR and high AR measurements and can be observed in the uncalibrated data. Figure 3 illustrates a nearly perfect -1 deg/deg slope phase vs. roll curve for a RH horn antenna with AR less than 0.5 dB, measured by a dual-linear range antenna with no calibration applied.

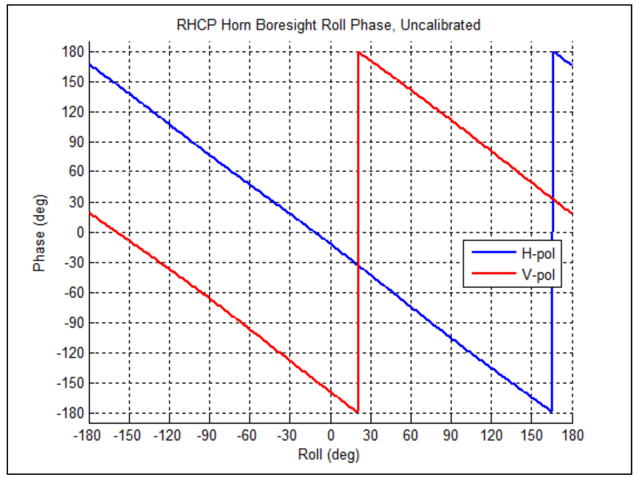

Figure 3 – Boresight Roll Phase for a RH AUT with AR < 0.5 dB

An inspection of Figure 1 confirms that a clockwise rotation of a RH AUT will produce decreasing phase curves similar to those in Figure 3. Figure 4 illustrates the stepped phase curves characteristic of a SGH boresight roll measured with a dual-linear range antenna, a measurement configuration that represents the special case described near the end of the discussion on Method A_1.

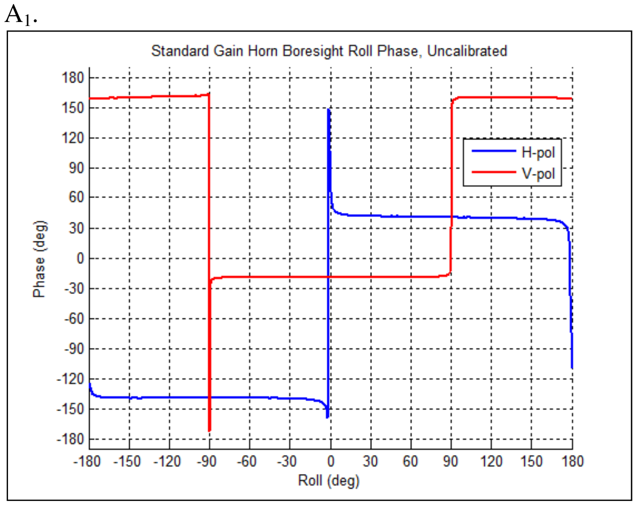

Figure 4 – Boresight Roll Phase for a SGH

Careful observation of the phase behavior in Figure 4 near the transitions illustrate that for increasing roll the H-pol curve trends slightly downward while the V-pol curve trends slightly upward. These slight trends are manifestations of slight right- and left-handedness, respectively, and are almost certainly characteristics of

[3]Positive roll (ϕ) is assumed to be clockwise rotation of a boresight AUT when viewed from the range antenna.

the test zone illumination rather than the SGH itself since the SGH cannot be both RH and LH simultaneously. Note that the relative phase between the H-pol and V-pol curves in both Figure 3 and Figure 4 is meaningless because calibration has not been applied. See the Appendix for more detail on the theory behind this sense verification technique.

7. Practical Considerations

Many references exist for quantifying gain, axial ratio, and tilt errors associated with the style of measurements described in this paper. Instead of revisiting these topics, we discuss more practical errors we have observed in industry and ways to avoid common mistakes associated with the process of converting orthogonal linear measurements into RH and LH circular components.

As mentioned previously, many publications assume the conjugate time convention of $e^{-i\omega t}$ and corresponding radial phase factor of e^{+jkr}. Some notable examples are [4, 5, and 8]. In cases where it is desirable to use formulas derived with the $e^{-i\omega t}$ convention assumed there are two valid approaches:

1. Swap –i for +j in all formulas [1 p. 55].
2. Conjugate the input data, use the formulas as-is, and then conjugate the results.

We have found that we make fewer mistakes by preserving the published formulas and using data conjugation as needed.

The most effective way to determine the time convention of a phase-coherent antenna measurement receiver is by increasing the range length by less than 1 free-space wavelength, preferably in many sub-wavelength steps. With increasing distance a decreasing phase indicates an $e^{-jkr}e^{+jwt}$ convention while an increasing phase indicates an $e^{+jkr}e^{-iwt}$ convention. While it is possible to deduce the receiver time convention by observing the uncalibrated phase slope vs. frequency this method is more prone to mistakes due to undersampling in frequency and incorrect assumptions about the relative lengths of the test and reference paths.

Every antenna measurement consists of the polarization product of both antennas involved until polarization correction is applied. Errors due to imperfect range antenna polarization can be estimated using information in [5] or corrected using methods described by [4, 8, 9]. In addition to polarization correction, more robust calibrations than those presented here can be performed using the port ratio technique described in [8 pp. 161-162].

Flexing cable/rotary joint errors affect both Methods during calibration, and can also affect Method A_1 measurements if the errors are not repeatable between the calibration and test.

Maintaining the calibration antenna in the same plane during the calibration procedure is critical to maintain proper phase reference between orthogonal polarization measurements. “Re-peaking” the SGH for maximum response between the two orthogonal calibration measurements is strongly discouraged. The phase error of seemingly small range length differences can become significant as frequency increases. Range antenna-to-AUT RF path length differences between the two orthogonal measurements must also be minimized, which can be very difficult for outdoor Method A_1 measurements due to system temperature drift and wind effects. While Method A_2 measurements are certainly more immune to these types of errors, wind can also affect electronically switched orthogonal measurements in two significant ways:

1. Older antenna measurement systems that place the frequency sweep on the innermost measurement loop may allow several seconds to pass between orthogonal measurements in some cases, allowing the wind to change the RF path length and corrupt the phase relationship between the measurements.
2. The multiple roll positions required for calibration with a SGH cannot typically be captured fast enough to avoid corruption by wind. One possible solution to overcome this predominantly phase-only error is to capture an additional calibration measurement with the SGH in a slant-45 orientation and equalize the system phase with the simultaneously-measured phase offsets. This technique assumes good linearity of the source antenna and SGH such that the phase transitions in Figure 4 trend toward pure steps with no slope elsewhere.

8. Summary

This paper presented several practical details that must be considered to properly convert orthogonal linear complex measurements into the circularly-polarized components necessary for characterizing the axial ratio, tilt angle, and sense of an AUT. Particular attention was given to the orientations of the antenna range polarization reference vectors, which are set during calibration but often misunderstood in industry. A step-by-step procedure was given along with a method to verify the results and insure the right-hand and left-hand polarization components have not been inadvertently swapped during the calculations.

9. Appendix

The theoretical basis for the technique we have used to determine the sense of a CP wave is based upon the transmission equation of spherical near-field scanning [8], once simplified to isolate the dependence of the relative received voltage, b_0'/a_0 upon the polarization angle χ_0 of the range antenna:

$$\frac{b_0'(kr_0;\varphi_0,\theta_0,\chi_0)}{a_0} = \sum_{\mu}\sum_{\substack{smn\\ \sigma\nu}} R'_{\sigma\mu\nu} C^{sn}_{\sigma\mu\nu}(kr_0\hat{z}) D^{(n)}_{\mu m}(\varphi_0,\theta_0,\chi_0) T_{smn}$$

$$= \sum_{\substack{\mu=\\ -1,+1}} \mathrm{T}_{\mu} e^{i\mu\chi_0}, \quad \text{where}$$

$$\mathrm{T}_{\mu}(kr_0;\theta_0,\phi_0) \equiv \sum_{\substack{smn\\ \sigma\nu}} R'_{\sigma\mu\nu} C^{sn}_{\sigma\mu\nu}(kr_0\hat{z})[d^{(n)}_{\mu m}(\theta_0)e^{im\phi_0}]T_{smn} \quad \text{(A1)}$$

The usual definitions apply: $R_{\sigma\mu\nu}$ is the receiving characteristic of the range antenna, T_{smn} is the transmitting characteristic of the test antenna, $C^{sn}_{\sigma\mu\nu}(kr_0\hat{z})$ is the translation matrix for spherical modes and $D^{(n)}_{\mu m}(\varphi_0,\theta_0,\chi_0)$ is the rotation matrix for spherical modes, written in terms of the Eulerian angles, ϕ_0, θ_0, χ_0 that describe the succession of three rotations which convert between the coordinates of the antennas. Parameter r_0 is the range length; k is $2\pi/\lambda$.

$$D^{(n)}_{\mu m}(\varphi_0,\theta_0,\chi_0) = e^{i\mu\chi_0} d^{(n)}_{\mu m}(\theta_0) e^{im\phi_0} \quad \text{(A2)}$$

The critical parameter here is the rotary angle χ_0. The far-field dependence of the range antenna's response as a function of polarization angle always takes the mathematical form of (A1). When the range receiving antenna is an elemental dipole, its relative voltage response is proportional to the electric field of the test antenna; so (A1) can be rewritten as describing the test antenna's electric field and its time dependence at a distance r_0, for aspect (or pattern) angles ϕ_0, θ_0, and for range polarization angle χ_0:

$$\overline{E}(kr_0,\varphi_0,\theta_0;\chi_0) = \mathrm{Re}[\sum_{\substack{\mu=\\ -1,+1}} \mathrm{T}_{\mu} e^{i\mu\chi_0} e^{-i\omega t}] \quad \text{(A3)}$$

Such an ideal range antenna is described by the receiving coefficients

$$R^{dip}_{\sigma\mu\nu} = \frac{\sqrt{2}}{2}\delta_{\sigma,2}\delta_{\nu,1}\left[\delta_{\mu,1} - \delta_{\mu,-1}\right] \quad \text{(A4)}$$

A circularly polarized electric field having RH sense and unit amplitude is described by the following parameters:

$$\mathrm{T}_{+1} = +1 \qquad \mathrm{T}_{-1} = 0 \quad \text{(A5)}$$

Setting $\chi_0 = \chi$ gives the x-component, and setting $\chi_0 = \chi + \pi/2$ gives the y-component of the AUT electric field,

$$\overline{E}(kr_0,\varphi_0,\theta_0;\chi_0) = \cos(\omega t - \chi)\hat{x} + \sin(\omega t - \chi)\hat{y} \quad \text{(A6)}$$

in agreement for χ=0 with equation 3.4 of [6]. Notice that the phase χ for RH sense becomes more negative for increasing values of range polarization angle χ_0. For LH sense, the phase becomes more positive.

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