An Algorithm for Automated Phase Center Determination and its Implementation.

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ABSTRACT

An efficient algorithm for calculating the position of the phase center of an antenna from a measurement is derived and implemented in software. Application of the algorithm to actual measurements shows that the success of the algorithm depends on characteristics of the antenna and a weighing parameter derived from the amplitude pattern.

Keywords: Phase center

1.0 Introduction.

The phase center of an antenna is a reference point such that the phase in the far field does not change when the antenna is rotated around this point in theta (angle from the Z-axis) or phi (angle around the Z-axis) direction. The phase center is often not uniquely defined, as most antennas are astigmatic, meaning that the phase center depends on the plane of observation. Still, for some applications it is a useful concept and in such cases it is important to know the location of the phase center, e.g. for illuminators of a reflector antenna where the phase center of the illuminator needs to be in the focal point of the paraboloid [1].

Measurement of the location of the phase center is often done using a mechanical construction that enables the operator to move the phase center to the center of rotation of the antenna measurement positioner, [2]. This requires interpretation of phase-plots of the antenna measurement. This paper presents an algorithm for easily estimating the location of the antennas phase center in a specific plane in cases where the concept of a phase center is useful and desired. An implementation in a script for NSI2000 [3] is demonstrated, together with results and limitations of the algorithm.

2.0 Determination of the offset between the phase center of the AUT with respect to the virtual rotation center.

Consider the model as drawn in Figure 1 where a point source (or Antenna Under Test, AUT) with unity amplitude rotates around a theta axis. There is a Z-offset dZ and an X-offset dX and the polarization is parallel to the axis of rotation. The angle of rotation is θ.

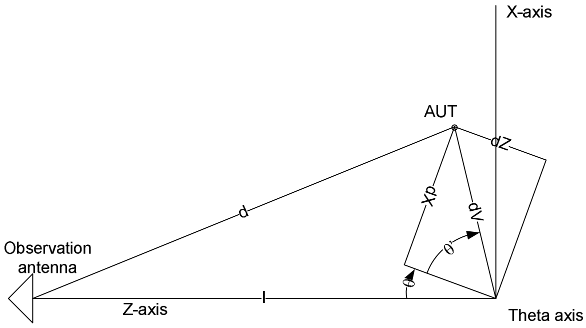

Figure 1: Model for determining the phase center

Consider an observation point at a distance l from the axis of rotation, lying on the Z-axis. Let dV be the radial distance of the AUT from the axis of rotation, where θ\` is

$$\theta' = \arccos\left(\frac{dZ}{dV}\right) = \arcsin\left(\frac{dX}{dV}\right) \quad (1).$$

Given this geometry, the distance between the AUT and the observation antenna is d, such that the signal produced at the observation point is

$$E(\theta) = \frac{e^{j \cdot k_0 d(\theta)}}{d(\theta)} \quad (2).$$

The distance d can be calculated as

$$d(\theta) = \sqrt{l^2 + dV^2 - 2 \cdot l \cdot dV \cdot \cos(\theta + \theta')} \quad (3),$$

and as such

$$E(\theta) = \frac{e^{j \cdot k_0 (\sqrt{l^2 + dV^2 - 2 \cdot l \cdot dV \cdot \cos(\theta+\theta')})}}{\sqrt{l^2 + dV^2 - 2 \cdot l \cdot dV \cdot \cos(\theta + \theta')}} \quad (4).$$

If we consider the case where the observer is in the far-field, the offset distance dV is negligible with respect to l. Eq (4) then reduces to

$$E(\theta) = \frac{e^{j \cdot k_0 (l - dV \cdot \cos(\theta+\theta'))}}{l} \quad (5),$$

where we use the following reduction

$$\lim_{l\to\infty}(l^2 + dV^2 - 2\cdot l\cdot dV\cdot\cos(\theta+\theta')) =$$
$$= \lim_{l\to\infty}(l^2 + (dV\cdot\cos(\theta+\theta'))^2 - 2\cdot l\cdot dV\cdot\cos(\theta+\theta')) =$$
$$= (l - dV\cdot\cos(\theta+\theta'))^2 \quad (6).$$

This allows us to write the phase φ(θ) as

$$\varphi(\theta) = k_0(l - dV\cdot\cos(\theta+\theta')) = \\ = \varphi_0 - k_0\cdot dV\cdot\cos(\theta+\theta') \quad (7).$$

This can be further simplified by using the identity

$$\cos(\theta+\theta') = \cos(\theta)\cdot\cos(\theta') - \sin(\theta)\cdot\sin(\theta') \quad (8)$$

and (1), resulting in

$$\varphi(\theta) = \varphi_0 - k_0\cdot(dZ\cdot\cos(\theta) - dX\cdot\sin(\theta)) \quad (9).$$

In order to find the position of the phase center of a real antenna under test, the measured phase is compared with the model as described above. To do so, consider the measured phase φ_i at the measurement angles θ_i.
The values for dX, dZ and φ_0 are determined by the values that minimize the Sum of the Squares values of the Difference (SSD) between measured and predicted values based on this model:

$$SSD = \sum_i(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))^2 \quad (10)$$

Minimizing for dX, dZ and φ_0 results in

$$\frac{dSSD}{d(dX)} = \frac{d\sum_i(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))^2}{d(dX)} =$$
$$= \sum_i\left(2\cdot(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))\cdot -k_0\cdot\sin(\theta_i)\right) = 0$$
$$\frac{dSSD}{d(dZ)} = \frac{d\sum_i(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))^2}{d(dZ)} =$$
$$= \sum_i\left(2\cdot(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))\cdot k_0\cdot\cos(\theta_i)\right) = 0$$
$$\frac{dSSD}{d(\varphi_0)} = \frac{d\sum_i(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))^2}{d(\varphi_0)} =$$
$$= \sum_i\left(2\cdot(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))\cdot -1\right) = 0 \quad (11)$$

This results in the following system of equations:

$$\begin{pmatrix} -k_0\cdot\sum_i\sin^2(\theta_i) & k_0\cdot\sum_i\sin(\theta_i)\cdot\cos(\theta_i) & -\sum_i\sin(\theta_i) \\ -k_0\cdot\sum_i\sin(\theta_i)\cdot\cos(\theta_i) & k_0\cdot\sum_i\cos^2(\theta_i) & -\sum_i\cos(\theta_i) \\ -k_0\cdot\sum_i\sin(\theta_i) & k_0\cdot\sum_i\cos(\theta_i) & -n \end{pmatrix}\cdot\begin{pmatrix} dX \\ dZ \\ \varphi_0 \end{pmatrix} = \begin{pmatrix} -\sum_i\varphi_i\cdot\sin(\theta_i) \\ -\sum_i\varphi_i\cdot\cos(\theta_i) \\ -\sum_i\varphi_i \end{pmatrix} \quad (12),$$

where n is the number of samples. This system is easily solved using gaussian elimination or Cramer's rule.

It must be noted that this algorithm uses the measured phase as a measure of distance between the AUT and the observation antenna. It is therefore required to unwrap the phase before the calculations can be performed.

3.0 Introduction of weighing factors.

The derived function has the property of assigning equal importance to each phase sample. However, it is considered that in practice we may want to emphasize the phase behaviour in areas where the amplitude is high, and de-emphasize the phase behaviour in areas where the amplitude is low, or even ignore the phase for amplitudes below a certain threshold. This reflects the fact that usually the direction of interest is the direction in which the gain of the antenna is high (of course, this depends on the actual application of the antenna under consideration).
In order to introduce such a weighing factor, equation (6) is modified:

$$SSD = \sum_i w_i\cdot(\varphi_i - (\varphi_0 - k_0\cdot(dZ\cdot\cos(\theta_i) - dX\cdot\sin(\theta_i))))^2 \quad (13).$$

Without repeating the intermediate steps, minimizing SSD for dX, dZ and φ_0 results in in the following system of equations:

$$\begin{pmatrix} -k_0 \cdot \sum_i w_i \cdot \sin^2(\theta_i) & k_0 \cdot \sum_i w_i \cdot \sin(\theta_i) \cdot \cos(\theta_i) & -\sum_i w_i \cdot \sin(\theta_i) \\ -k_0 \cdot \sum_i w_i \cdot \sin(\theta_i) \cdot \cos(\theta_i) & k_0 \cdot \sum_i w_i \cdot \cos^2(\theta_i) & -\sum_i w_i \cdot \cos(\theta_i) \\ -k_0 \cdot \sum_i w_i \cdot \sin(\theta_i) & k_0 \cdot \sum_i w_i \cdot \cos(\theta_i) & -\sum_i w_i \end{pmatrix} \cdot \begin{pmatrix} dX \\ dZ \\ \varphi_0 \end{pmatrix} = \begin{pmatrix} -\sum_i w_i \cdot \varphi_i \cdot \sin(\theta_i) \\ -\sum_i w_i \cdot \varphi_i \cdot \cos(\theta_i) \\ -\sum_i w_i \cdot \varphi_i \end{pmatrix} \quad (14)$$

Note that with setting the weighing factor to 1, equation (14) reduces to equation (12).

Typical examples of appropriate weighing factors are the amplitude that corresponds to the measured phase (expressed in normalized power, not in dB's) or a threshold function based on the measured power so that only the phase in a main-lobe determines the location of the phase center.

4.0 Compensation of a phase center offset.

To verify the quality of the calculations of the phase center position, it is desirable to compensate for the phase center offset. A constant phase over a certain region would indicate that the calculation is indeed successful. To do so, a θ_i dependent phase needs to be added to the measured phase. This compensation term ψ_i is given as

$$\psi_i = -k_0 \cdot \left(dX \cdot \sin(\theta_i) + dZ \cdot \cos(\theta_i) \right) \quad (15).$$

5.0 Practical implementation and application.

The described phase center position calculation and compensation algorithm has been implemented in an NSI2000 script. In order to demonstrate the performance of the algorithm and to investigate the influence of different weighing schemes, phase center calculations and compensations were performed on spherical near-field measurements on two different antennas: a WR430 standard gain horn measured at 2.4 GHz and a dual linear log periodic antenna measured at 9 GHz. The results are shown in Figure 2 thru Figure 9. Figure 2 and Figure 4 show the amplitude diagrams in azimuth and elevation for the WR430 SGH, respectively. These amplitude patterns are used in the weighing of the phase during the determination of the location of the phase center.
Figure 3 and Figure 5 show the corresponding phase diagrams in azimuth and elevation. Note that in order to determine the phase center, independent calculations are performed for the azimuth and elevation planes. Each of the phase diagrams show the phase without phase center compensation (red curve) and the phase after compensation for a phase center calculated using different weighing schemes: the blue curve shows the compensated phase using no weighing, the purple curve using the amplitude as weighing factor and the green curve using a threshold weighing. In the latter situation, in the calculation of the phase center, only the phase was used for samples where the amplitude was less then 10 dB below the maximum amplitude.
Figure 6 and Figure 8 show the amplitude (azimuth and elevation) for the dual linear log periodic antenna, while Figure 7 and Figure 9 show the corresponding phase diagrams.

In order to determine whether the phase center compensation is successful, the phase curves are observed. In the optimal case, the phase curve after compensation is flat and horizontal within the angular range of interest (usually a region where the gain is high). A convex or concave curve for the compensated phase indicates that the offset in the Z-direction is not optimally compensated. If the non-compensated phase curve was convex and the compensated phase curve is convex too, the compensation was incomplete (under-compensated) in Z. If the phase curve is now concave, the phase was overcompensated in Z. The reverse is true for an originally concave phase curve.
A slope in the curve for the compensated phase indicates that the offset in the X-direction is not optimally compensated. If the slope is in the same direction as the uncompensated phase, the phase compensation was incomplete (under-compensated) in X, while a slope in the opposite direction of the original curve indicates overcompensation in X.

From the azimuth phase diagrams of the pyramidal horn (Figure 3) it is observed that by using the phase center position that is calculated using amplitude weighing (purple curve), the most optimal result is obtained. In the area that can be considered the beam peak (amplitude less than 10dB below maximum), the phase curve is flattest of the shown curves. Te curve is not perfectly flat as the antenna itself does not have the ideal phase behavior as described in our model, but the deviation from the ideal flat curve is the least of the compensated curves. The phase curve using no weighing (blue curve) is under-compensated, while the curve using threshold weighing (green curve) is overcompensated.

The elevation phase diagrams (Figure 5) of the pyramidal horn show that in that plane, the calculation of the phase center without weighing for compensating the phase diagram (the blue curve) produces optimal results. The use of amplitude (purple curve) or threshold (green curve) weighing causes overcompensation: the uncompensated curve (red) is convex, the curve using compensation without weighing is flat in the area of the main-beam and the curves using amplitude or threshold weighing are concave.

Observing the phase diagrams of the log periodic antenna (Figure 7 and Figure 9), the best results (flat and horizontal phase diagram) are obtained using threshold weighing.
In the azimuth direction, the blue curve (no weighing) shows incomplete compensation in the X-direction resulting in a slope that remains negative, while there is a slight overcompensation in the Z-direction. Although the purple curve (amplitude weighing) is horizontal, it is still convex, and as such the compensation in the Z-direction is incomplete.
In the elevation direction, both the blue (no weighing) and purple (amplitude weighing) curve are convex, indicating under-compensation in the Z-direction. Both the blue and purple curves have a slope opposite of the uncompensated curve, meaning that the phase center is overcompensated.

From the results as tabulated in Table 1 and Table 2 it is readily observed that the Z- offset is not uniquely defined for an antenna, stressing the statement about astigmatism in the introduction. The difference in offset in the X-direction between azimuth and elevation patterns is of course due to the different planes in which the offset is determined: the X-offset calculated from an azimuth cut is a horizontal displacement, while the X-offset calculated form an elevation cut is a vertical displacement.

	Azimuth offset (in degrees of phase)		Elevation offset (in degrees of phase)	
	dX	dZ	dX	dZ
No weighing	6.02	-323.85	7.06*	-332.09*
Amplitude weighing	6.08*	-488.44*	5.39	-473.31
Threshold weighing	6.51	-517.87	8.50	-481.81

Table 1: Calculated offset in dX and dZ direction of a pyramidal horn at 2.4 GHz (* indicates best result).

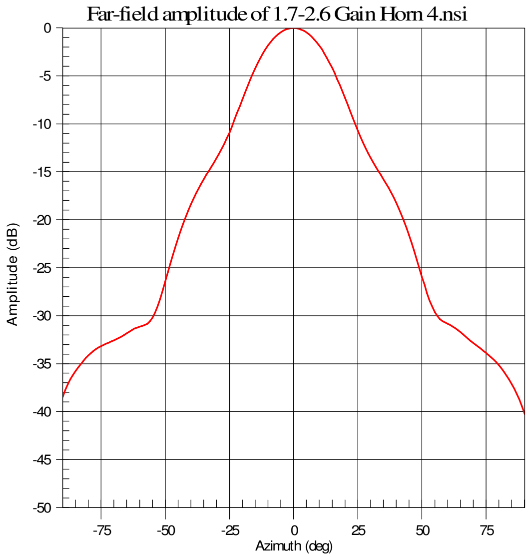

Figure 2: Azimuth amplitude pattern of pyramidal horn at 2.4 GHz.

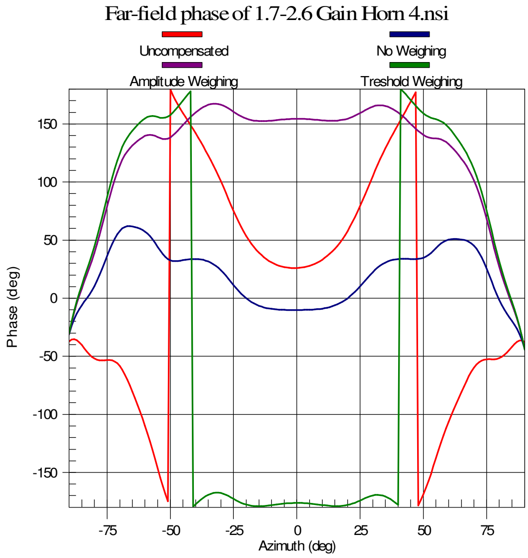

Figure 3: Azimuth phase pattern of pyramidal horn at 2.4 GHz before and after compensation for phase center offset.

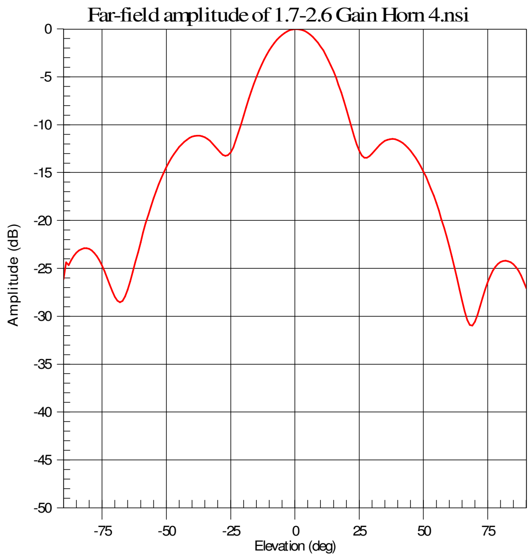

Figure 4: Elevation amplitude pattern of pyramidal horn at 2.4 GHz.

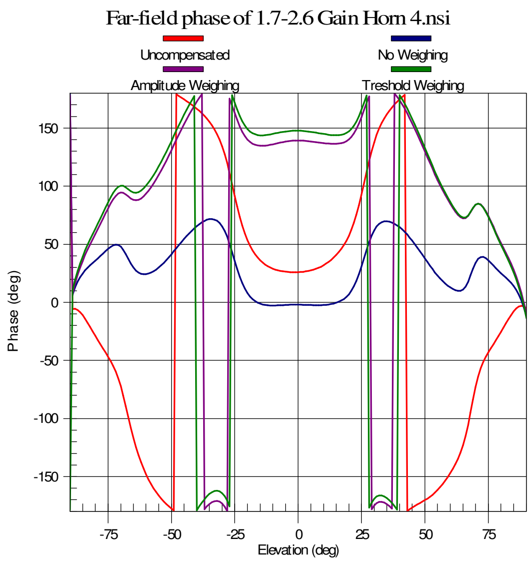

Figure 5: Elevation phase pattern of pyramidal horn at 2.4 GHz before and after compensation for phase center offset.

	Azimuth offset (in degrees of phase)		Elevation offset (in degrees of phase)	
	dX	dZ	dX	dZ
No weighing	2.38	-29.94	15.56	50.25
Amplitude weighing	-10.59	31.56	10.59	25.29
Threshold weighing	-11.35*	-10.16*	7.89*	15.56*

Table 2: Calculated offset in dX and dZ direction of a dual linear log periodic antenna at 9 GHz (* indicates best result).

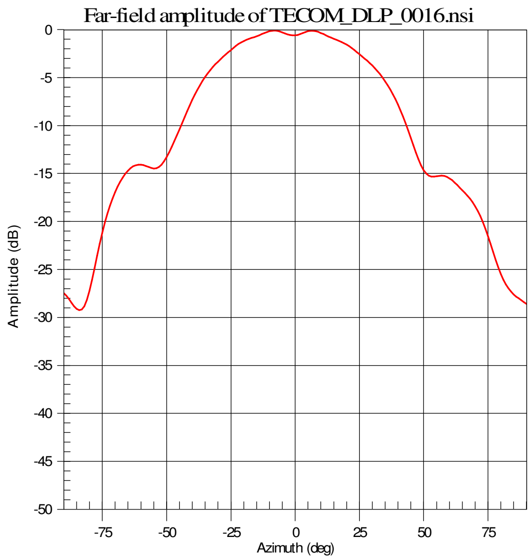

Figure 6: Azimuth amplitude pattern of dual linear log periodic antenna at 9 GHz.

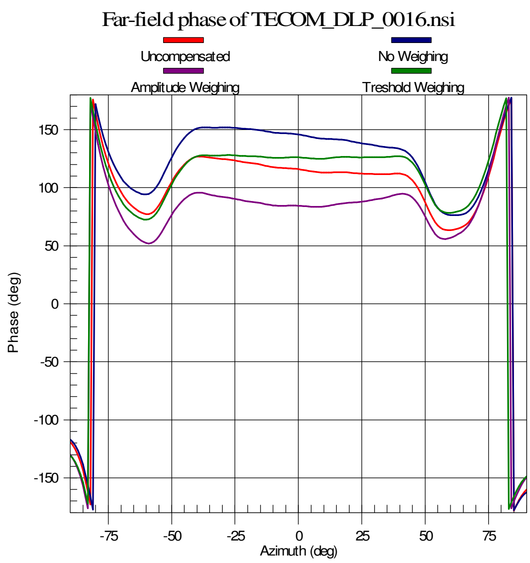

Figure 7: Azimuth phase pattern dual linear log periodic antenna at 9 GHz before and after compensation for phase center offset.

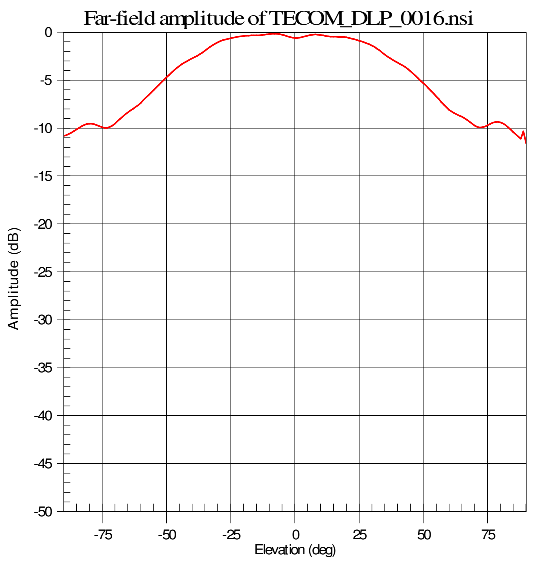

Figure 8: Elevation amplitude pattern of dual linear log periodic antenna at 9 GHz.

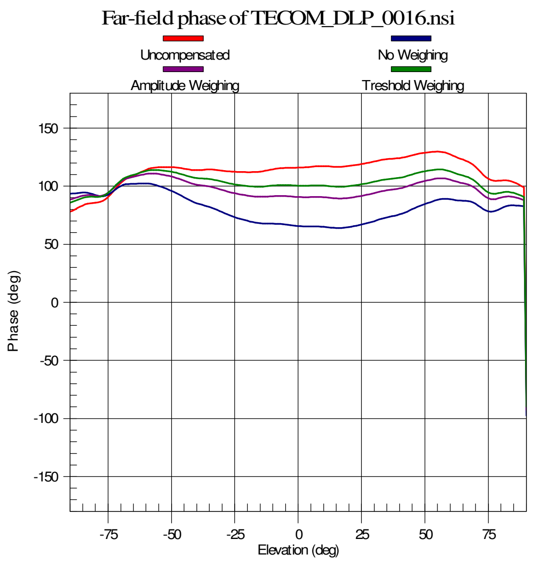

Figure 9: Elevation phase pattern dual linear log periodic antenna at 9 GHz before and after compensation for phase center offset.

6.0 Conclusion and recommendations.

An efficient algorithm to determine the position of the phase center from an amplitude/phase measurement of an antenna is presented. From applying the algorithm to actual measurements, it is observed that the actual results depend on the characteristics of the antenna and the applied weighing.

As the Z-position of the phase center depends on the observed measurement plane, it may be useful to enhance the algorithm to observe multiple planes and derive an overall value for the Z-position from the individual plane measurements. This can be done using averaging or using a full-sphere approach.

Considering the model described to match the measured phase in order to find the phase center position, it is clear that this approach only is applicable for antennas with a single main lobe in which the phase is more or less constant. Antennas with multiple lobes with different phases (like monopulse antennas) do not obey this model and the algorithm will fail to work accurately. A pre-processing algorithm (like changing phase by 180° in adjacent lobes) may solve this issue.

7.0 REFERENCES

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