

THE USE OF STATISTICAL IMAGE CLASSIFICATION AS A CONVERGENCE CRITERIA IN PHASELESS NEAR-FIELD ANTENNA MEASUREMENTS

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ABSTRACT

The utility of a variety of objective, quantitative and robust methods of assessing similarities between antenna measurement data have already been highlighted in the literature. These techniques essentially involved the extraction of interval, ordinal, and categorical features from antenna pattern data sets that can then be effectively compared to establish a measure of their adjacency, *i.e.* similarity. Hitherto, such techniques have primarily been limited to the purposes of comparing two or more images as a means in itself, *e.g.* far-field three-dimensional radiation pattern of a given antenna having been characterised using two different facilities. In contrast, this paper discusses the utility of such techniques for the purposes of establishing convergence within an iterative optimisation process, namely the phase retrieval (PR) plane-to-plane algorithm. Within this paper, in addition to the conventional holistic error metrics, an alternative image classification comparison technique is employed as an error metric. The convergence properties, as reported by these various metrics are compared and contrasted using empirical mm-wave measured data taken using a planar near-field scanner and processed using a commonly encountered plane-to-plane PR algorithm.

Keywords: Image Classification, Phase-less, Near-field, Antenna Measurements.

1.0 Introduction

Phaseless near-field antenna measurements, together with the requisite processing, constitutes a valuable tool in the metrologists arsenal. Conventionally, near-field antenna measurements require that highly accurate measurements are taken of the amplitude and phase of the propagating portion of two orthogonal tangential near electric field components in the vicinity of the radiator. However, PR

from intensity only measurements at two near-field surfaces is an attractive, well known, technique for obtaining asymptotic far-field parameters when access to coherent holographic, *i.e.* amplitude *and* phase, data is either inconvenient or impossible. An inability to accurately measure phase can arise when attempting to characterise radiators operating at very high frequencies. This can result either from limitations within the RF or mechanical positioning sub-systems, or when the cost of such specialist equipment becomes prohibitive.

When employing iterative or minimising algorithms such as these, error metrics are generally employed to monitor and assess the progress of the algorithm in question. These metrics, or error functions, are used to assess parameters such as convergence rate, which is the change in the error metric per iteration, and saturation level, which is the level obtained when the value of the error function remains unchanged after each successive iteration, become the focus of attention. However, this intermediate assessment task is complicated by the volume, complexity and large dynamic range of the data concerned which crucially, can effect the sensitivity, robustness, and ultimately the utility of the convergence criteria.

2.0 Levels of Measurement

In any representational theory of measurement, when a measurement is made of any physical phenomena, conventionally different levels of information can be defined to exist within the measurement data. Thus, the measurements can be defined as nominal, ordinal, interval or ratio dependant on the level of information that is extracted in the measurement process.

In nominal measurements the results of the measurement procedure are nominated or assigned to specific categories, *e.g.* an antenna could as a result of a

measurement process be categorised as high gain or low gain. So the value of the measurement result is that of the set it is assigned to, where this set is defined in intension by the property by which measurement results are to be categorised, *e.g.* the sets defined by intension as high gain or low gain antennas. The only kind of measure of central tendency that can be applied is modal and although dispersion about this mode can be approximated by a variety of indexes of qualitative variation, for nominal measurements the categorisation is the only significance of the measurement.

In ordinal measurements the numbers assigned to represent the measurand are rank orders. Thus the levels of measurement defining the variable are ranked or ordered where comparisons of greater than, less than or equality are viable. The central tendency of the data set can be represented by the median and this along with a range of other statistics, *e.g.* percentiles, can be used to define significant measures in any measurement result.

In interval measurements the numbers assigned to objects have all the features of ordinal measurements, and in addition equal differences between measurements represent equivalent intervals, *i.e.* the logical form of the representation is quantitatively the same as the logical form of the measured variable that is the measurand. This means that certain arithmetic operations between, and on, data points in the measurement results are valid. Thus, a range of techniques can be used to produce significant statistics, *e.g.* arithmetic mean, for any sampled data set like that acquired in a near-field scanner.

Finally, ratio measurements can be made which have all the features of interval measurement but also have meaningful ratios between measurement results and standards or defined units, thus operations such as multiplication and division are always meaningful. Unlike an interval scale, the zero-value on a ratio or metric scale is non-arbitrary and this allows the use of many different techniques to establish a range of significant measures between data sets.

Clearly the information required to represent the any measurement result on these nominal, ordinal or interval scales is contained within the ratio data. However if there is a requirement to analyse the data, the different aspects and levels of measurement represented on the different scales are amenable to very different statistical methodologies. As a result considerable insight into the nature of any measurement procedure can be obtained by an approach that does not limit itself to just examining the interval or ration information in the data.

3.0 Amplitude Only Measurements

The PR problem arises in applications of electromagnetic theory in which wave phase is apparently lost or is impractical to measure and only intensity data is available. The planar near-field methodology requires holographic measurements to be made. In other words, in order that the angular spectrum can be obtained and thence the asymptotic far-field, knowledge of both the amplitude and phase of the near-field must be available. However, direct measurement of phase becomes progressively more difficult as the frequencies concerned become higher. Matters are further compounded since even antennas that are comparatively modest in physical size, when operating at high frequencies, will constitute electrically large instruments.

Many alternatives are available for recovering the phase from amplitude only measurements,[1] with perhaps the most common being the use of multiple intensity distributions which permit the use of iterative computational procedures. An extensively described methodology for the acquisition of such phase less measurement data, using a plane-to-plane PR algorithm with an aperture constraint can be can be described as follows [2]:

1. Measure the amplitude of the field over plane 1.
2. Measure the amplitude of the field over plane 2.
3. Guess the amplitude and phase of the antenna aperture illumination function.
4. Truncate the fields to the physical extent of the antenna aperture.
5. Use PTP transform to propagate the AUT aperture fields to plane 1.
6. Replace the amplitude estimation at plane 1 with the measured amplitude at plane 1.
7. Use PTP transform to propagate the fields back to the AUT aperture plane.
8. Truncate the fields to the physical extent of the antenna aperture.
9. Use PTP transform to propagate the AUT aperture fields to plane 2.
10. Replace the amplitude estimation at plane 2 with the measured amplitude at plane 2.
11. Use PTP transform to propagate the fields back to the AUT aperture plane.
12. Repeat steps 4 to 11 until amplitude on plane 1 (or plane 2) has converged to within a prescribed tolerance.
13. Transform the fields to the far-field using standard algorithm.

Figure 1: Plane-to-plane PR algorithm using aperture constraint.

The aperture constraint form of the plane-to-plane PR algorithm was employed here purely as when characterising antennas with a well defined aperture, this implementation converges more rapidly, in this case roughly 100 times faster, than some other more general implementations. An integral part of the process listed above as step 12 is, repetition of the process until

convergence to within a prescribed level. Conventionally only a limited range of adjacency measures based on the interval nature of the data have been used to assess the convergence of the process. However an examination of other levels of information in the phase less measurement data reveals other types of information, which could be used in the assessment of convergence, and suggests that other techniques may be applicable.

In order that the convergence properties of the plane-to-plane PR algorithm could be investigated, a high gain dual reflector antenna assembly, shown in Figure 2 below, was acquired using the Queen Mary, University of London (QMUL) NSI-200V-3x3 plane rectilinear near-field measurement system. Measurements were taken at 95 GHz over the surface of two parallel planes in the propagating near-field of the antenna under test (AUT) which were separated from the aperture plane by 105 mm and 235 mm respectively. The narrow beam-width of this antenna, *circa* 2°, resulting from the electrically large 254 mm diameter (~ 80 wavelengths) main reflector, together with comparatively the short range length and large acquisition window minimised the amount of truncation in the measurements. In this case, this equated to a first-order truncation angle for the plane with the greatest separation of approximately $\pm 44^\circ$ in both azimuth and elevation ($\sin 44^\circ \approx 0.7$).

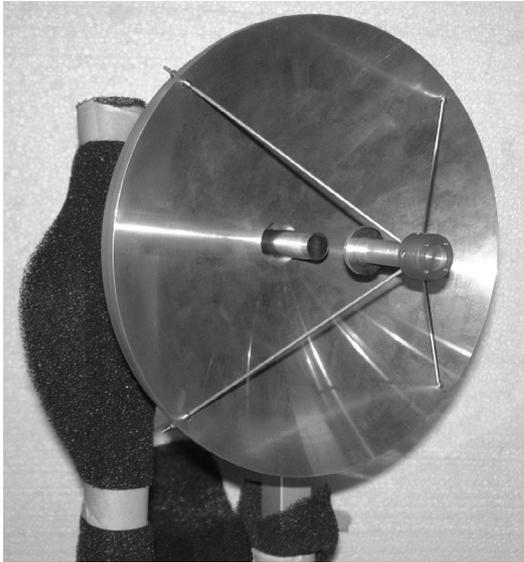


Figure 2: High Gain Dual Reflector Antenna Operating at mm-Wave Frequencies.

The algorithm outlined in Figure 1 above was harnessed to reconstruct the near-field phase information. This requires that the efficient fast Fourier transform (FFT) based plane-to-plane field propagation formula be utilised. Essentially, this states that if the field is known

over one plane, which can be defined arbitrarily to be at $z = 0$, then the field over another parallel plane can be expressed succinctly as,

$$u(x, y, z) = \mathfrak{F}^{-1}\{\mathfrak{F}\{u(x, y, z = 0)\}e^{-jk_z z}\} \quad (1)$$

Where \mathfrak{F} and \mathfrak{F}^{-1} denote the Fourier and inverse Fourier operators which are defined as,

$$\mathfrak{F}\{u(x, y, z = 0)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y, z = 0) e^{j(k_x x + k_y y)} dx dy \quad (2)$$

$$\mathfrak{F}^{-1}\{F(k_x, k_y) e^{-jk_z z}\} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_x, k_y) e^{-j(k_x x + k_y y + k_z z)} dk_x dk_y \quad (3)$$

Here, z is used to denote the amount by which the planar field distribution is to be propagated, u is the measured complex field, F is the equivalent angular spectrum, x, y are the spatial co-ordinates of the field point and k_x, k_y and k_z are the x, y , and z components of the propagation vector whilst j is the imaginary unit. Figure 3 and Figure 4 below contain cardinal cuts of the conventional holographic (blue and black trace) and phase recovered (dotted black trace) angular spectrum patterns plotted together with the equivalent multipath level (EMPL) (grey trace). The EMPL is half the absolute difference between the patterns expressed in dB [3]. Within this measurement campaign coherent holographic, *i.e.* amplitude *and* phase, data was acquired. The phase information was discarded for the purposes of testing the phase recovery algorithm however the far-field patterns obtained from the conventional coherent near-field data constitute an excellent means with which to assess the effectiveness of the PR algorithm.

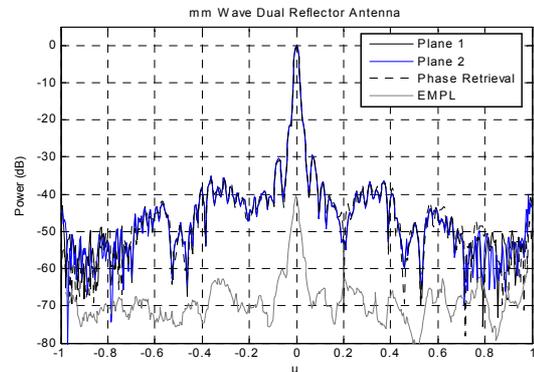


Figure 3: Far-Field Horizontal Pattern Cuts Obtained From Coherent and Amplitude Only Measurements.

From inspection, the degree of agreement attained between the respective sets of data can be seen to be very encouraging and this observation is further illustrated by the low values of the EMPL which are approximately 20 dB below the level of the patterns. It should be noted that an eleven point rolling boxcar average has been used to smooth the EMPL so that the underlying nature of the pattern is more easily seen. Although not show, similarly

encouraging results were attained for the vertical cardinal cut. The reconstructed aperture illumination function can be seen plotted in Figure 4 and Figure 5 below.

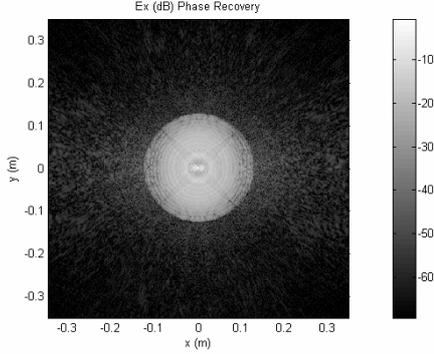


Figure 4: Amplitude Aperture Illumination Function

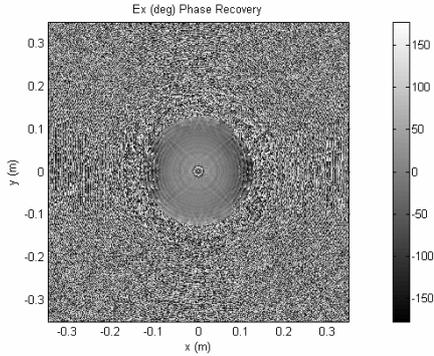


Figure 5: Phase Aperture Illumination Function

The reconstructed aperture illumination function further illustrates the success of the technique as the phase recovered results were found to be in close agreement with the phase measured results. It was also encouraging that effects resulting from the four sub-reflector support struts could be seen within the amplitude and phase patterns. However, the effect of the application of the aperture plane spatial filter could clearly be seen within the field outside the physical aperture as these have been suppressed which was most clearly seen in the horizontal plane.

4.0 Convergence Metrics

Within section 3 above, it was shown that amplitude only near-field measurements could be used to successfully determine the far-field pattern of a high gain aperture antenna by means of a highly efficient, iterative phase retrieval, algorithm. The aforementioned iterative loop was terminated manually after 200 passes, irrespective of the value of the convergence criteria at that point. The value of 200 was chosen, almost, arbitrarily from inspection of the data as a best compromise between computational effort and quality of results. However, at

the end of each cycle of the algorithm, the value of a number of different error metrics were computed and stored. Within the following section, a number of different convergence criteria, *i.e.* penalty functions, are presented and their results discussed.

4.1 Description of Conventional Assessment Technique

Conventionally, most PR algorithms have utilised at the sum squared difference (SSD) between the near-field patterns at the measurement plane, and usually the furthest measurement plane, as the error metric. This is computed by summing the squared difference of the calculated modulus of the field, $|u_{i,j}|$, and the measured modulus of the field, $m_{i,j}$, at each point (i,j) over the measurement plane. Mathematically, this can be expressed as [4],

$$\xi_{SSD} = \frac{\sum_{i,j} (|u_{i,j}| - m_{i,j})^2}{\sum_{i,j} m_{i,j}^2} \quad (4)$$

Although the SSD error metric, ξ_{SSD} , can be plotted in dB form within this paper, it will be plotted linearly so that it may be compared with other candidate error metrics more easily. Figure 6 below contains a plot of the SSD error metric. Here, it is evident that as the algorithm progresses the degree of agreement reported between the modulus of the predicted field and the modulus of the measured field generally improves. However, after the initial comparatively rapid conversion over the first few 20 or so iterations, the rate of convergence decreases in a near asymptotic form, *i.e.* the graph becomes level beyond ~ 100 iterations of the algorithm. From inspection of Figure 6, it is quite apparent that the SSD error function is both an effective, and fairly sensitive, measure of the convergence properties of the plane-to-plane PR algorithm and therefore fit for purpose. However, the question remains, can anything be done to improve things?

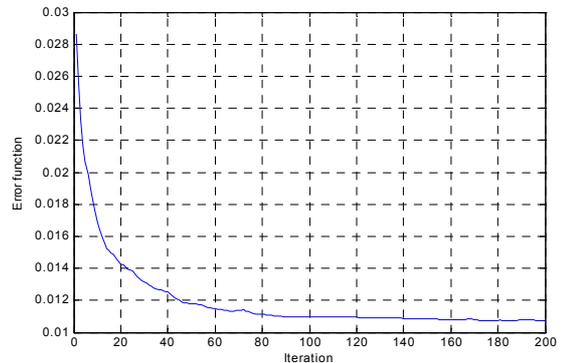


Figure 6: Sum Squared Difference Error Function.

Two phenomena are apparent from Figure 6; firstly the rate of change of the error function with respect to the iteration becomes progressively small as the number of iterations increases (*i.e.* the saturation level) and secondly, the improvement in the degree of agreement is not monotonic encountering local minima (*i.e.* stagnation). Thus, if the stopping criteria depends purely upon the error function attaining some pre-determined value which represents an absolute error limit, the asymptotic nature of the process may result in an excessively, possibly even infinitely, long processing time. Alternatively, if the stopping criteria depends purely upon an error convergence limit, *i.e.* essentially monitoring the rate of change of the error function, this could result in the algorithm stopping at a local minimum, rather than at the global minimum. This can perhaps be seen more clearly illustrated in Figure 7 below which contains a plot of the rate of change of the error function with respect to the iteration.

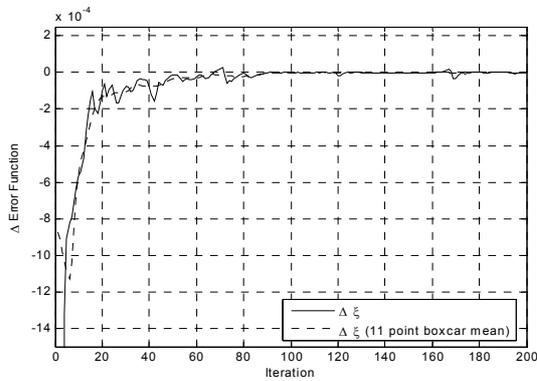


Figure 7: Slope of Sum Squared Difference Error Function.

Saturation can be seen as the plot tends towards zero and stagnation can be seen in the form of sharp fluctuations in the function. Figure 7 contains a second dotted trace which represents the numerical gradient of the error metric after it has been smoothed with a simple eleven point rolling “boxcar” mean average. This has the effect of suppressing rapid fluctuations in the error function with the intension of making the underlying trend of the function more apparent. After the first few iterations, the smoothed, *i.e.* filtered, error metric can be seen to stabilise and perhaps provide a more useful measure of the performance of the algorithm which is less prone to the influence of outlying points.

4.2 Alternative Error Assessment Techniques

Other conventional measures of correspondence, not so closely associated with antenna measurements, can also be used to assess the data. If two signals, such as antenna patterns, vary similarly point for point then a measure of

their similarity may also be obtained by taking the sum of the products of the corresponding pairs of points. If the two sequences of numbers are independent and random the sum of the products will tend to zero as the number of pairs of points is increased to infinity, as all numbers positive and negative are equally likely. If however, the sum is finite and non-zero this will indicate a degree of correlation. This is the basis for the cross-correlation function (CCF) which, in its normalised form, can be expressed mathematically as [3],

$$\xi_{CCF} = \frac{\sum_{i,j} I_1(i,j)I_2(i,j)}{\sqrt{\left(\sum_{i,j} I_1^2(i,j)\right) \cdot \left(\sum_{i,j} I_2^2(i,j)\right)}} \quad (5)$$

Here, I_1 would be assigned the value of calculated modulus of the field, $|u_{i,j}|$, and I_2 the measured modulus of the field, $m_{i,j}$ and ξ_{CCF} denotes the CCF. The CCF is normalised so that its value always lies in the range, $-1 \leq \xi_{CCF} \leq 1$ where +1 implies perfect correlation, 0 signifies no correlation and -1 represents opposite signals, *i.e.* signals out of phase by π . Figure 8 below contains a plot of the, un-smoothed, CCF plotted as a function of iteration.

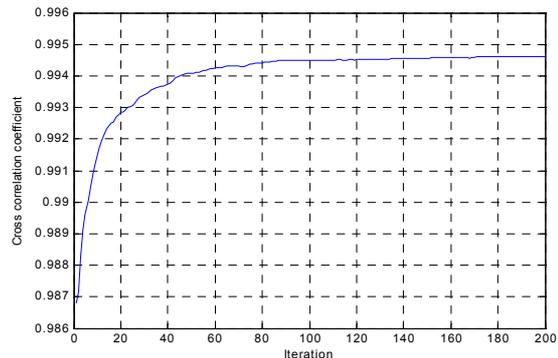


Figure 8: Cross-Correlation Error Function.

Many of the features of Figure 8 are in common with Figure 6 including the fact that the total variation in the value of the CCF is small. In practice, minor differences between otherwise similar patterns are not well discriminated and the CCF appears to provide no clear advantage over the conventional SSD metric. This is a commonly encountered problem when attempting to measure the adjacency between two or more antenna patterns and is often a problem when utilising interval assessment techniques. There are two specific aspects of the measurement methodology that can handicap any interval pattern assessment of antenna patterns produced by near-field scanning. These are, the very high dynamic range of the measurement system and, the interferometric nature of the measurement and the lack of uniformity of

the reference source. Although in this instance the interferometric nature of the data is, by definition, not a concern any lack of uniformity in the reference is. Put another way, any amplitude drift in the system between successive acquisitions or imperfections in alignment could complicate the interval assessment process.

As has been shown previously [3], an ordinal measure of association that can overcome this limitation can be derived if the interval nature of the data is temporarily ignored. A ranking is a permutation of integers that represents the relative ordering between interval values in a data set. If the ranks are not unique, *i.e.* two elements have the same value then the elements are ranked so that the relative spatial ordering between elements is preserved *i.e.* the first element has the smallest rank. The correlation between two rankings can be considered to constitute a measure of closeness, or distance between the two sets. A detailed description of this comparison process is beyond the scope of this paper but can be found within the literature [3, 5]. Figure 9 below contains a plot of the ordinal measure of adjacency (OMA), ξ_{OMA} , having been calculated after each iteration of the phase recovery algorithm.

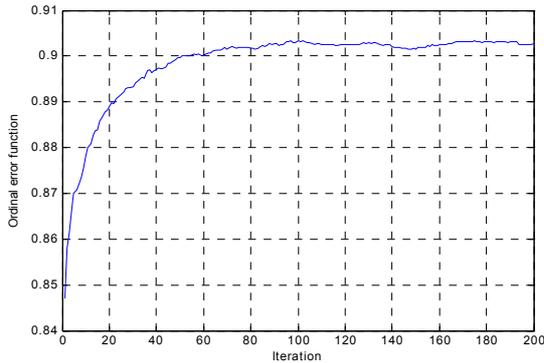


Figure 9: Ordinal Error Function.

Again, the OMA is normalised so that its value always lies in the range, $-1 \leq \xi_{OMA} \leq 1$ where +1 implies perfect correlation, 0 signifies no correlation and -1 represents opposite signals. A useful property of the OMA is that it provided a larger variation in value that was the case for either of the interval based alternatives, thus making further use of the results easier and more reliable.

5.0 Discussion and Conclusions

Despite the research reported within this paper is ongoing, this paper has confirmed that reliable far-field and aperture plane data can be obtained from amplitude only measurements taken at millimetre wave frequencies using a small planar scanner and an iterative plane-to-

plane phase retrieval algorithm. Within this measurement campaign a number of convergence criteria were compared and contrasted. This is the first time that such less commonly encountered measures of adjacency have been employed as a penalty functions within iterative phase-retrieval algorithms. Hitherto, an interval SSD data assessment approach has been adopted to determine the convergence criteria within the iterative plane-to-plane phase retrieval algorithm. It has been shown that such comparison techniques can prove to be problematic when assessing data sets which inherently have a high dynamic range, a large amount of data, and require high stability in its acquisition. In such circumstances, alternative statistical approaches have been found to be beneficial as they are not susceptible to these influencing factors. Within this paper an alternative measure based upon an ordinal assessment of the data has been demonstrated as being a viable alternative.

The ordinal measure demonstrated that the small but systematic errors introduced into the predictions can be accurately quantified in the calculation of the k value. Crucially, for the assessment of these results this metric provided a sensitive and stable measure of similarity. However, the ordinal process of ranking the data to produce permutations takes no account of either the absolute amplitude or spatial position at which the data is found. The robustness of this technique tends to render the method insensitive to smaller differences, particularly to “textures” within the pattern, *i.e.* to subtiles within the overall pattern. Crucially however, the variation in the ordinal error function can be seen to be significantly greater than any of the equivalent conventional interval error functions assessed thus allowing greater resolution in the assessment whilst simultaneously remaining robust to influencing factors that are inherent within the data sets being assessed.

Although other measures of adjacency exist, the authors were prevented from discussing these here due to the constraints of conciseness and time. Whilst it is clear that there is no “right” answer to the very general question of how best to compare two patterns, research is ongoing to establish which combinations of assessment techniques are best suited to the analysis of antenna patterns.

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